

RATIONAL FUNCTION - the quotient of two polynomials

$$f(x) = \frac{N(x)}{D(x)}$$

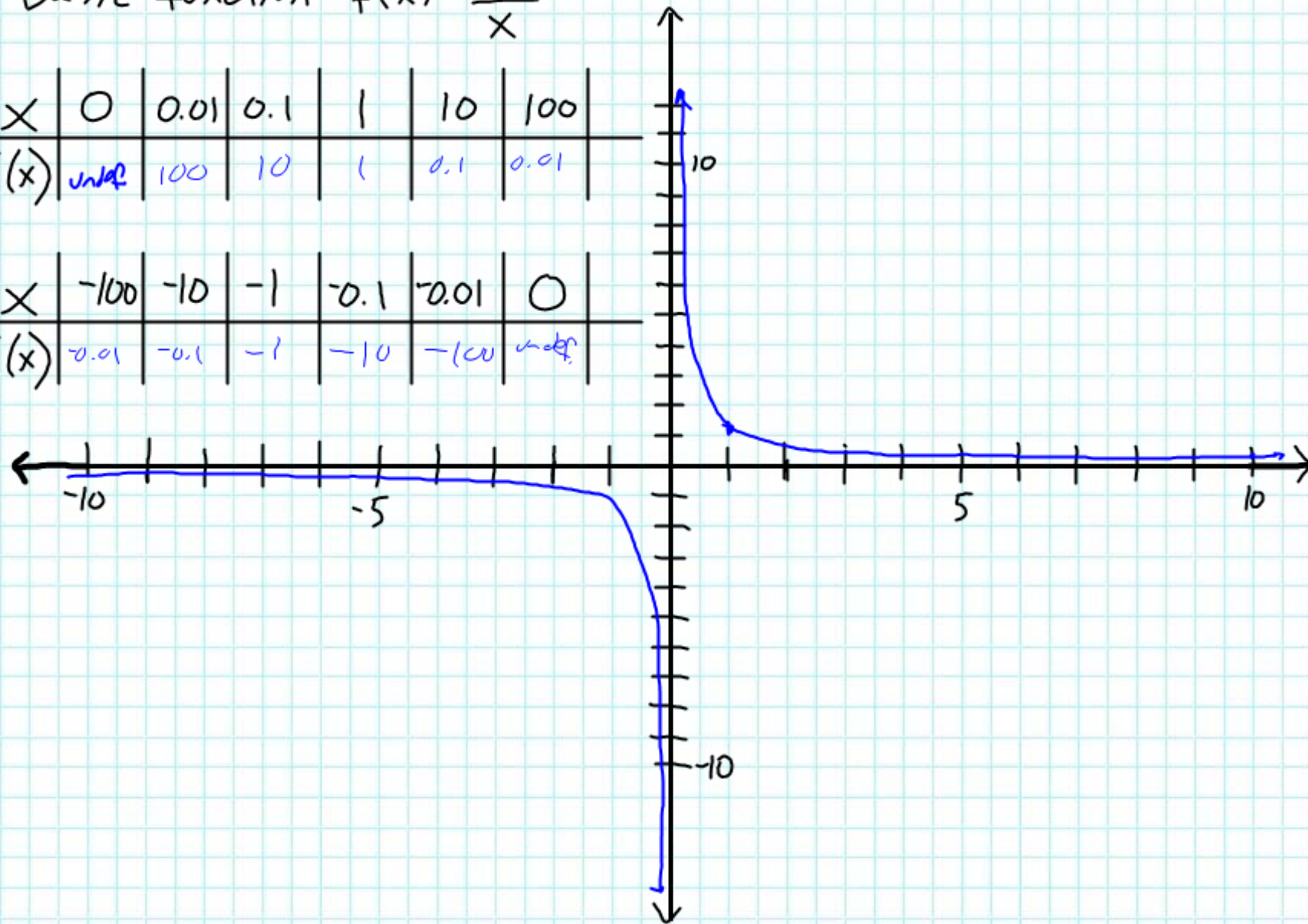
Domain

All  $x$ , such that  $D(x) \neq 0$

Basic function  $f(x) = \frac{1}{x}$

$x$	0	0.01	0.1	1	10	100
$f(x)$	undef	100	10	1	0.1	0.01

$x$	-100	-10	-1	-0.1	-0.01	0
$f(x)$	-0.01	-0.1	-1	-10	-100	undef



ASYMPTOTES

$$f(x) = \frac{N(x)}{D(x)} \quad \text{and } n = \text{degree of top polynomial}$$

$$m = \text{degree of bottom polynomial}$$

VERTICAL ASYMPTOTES — occur when  $D(x) = 0$

E.g.  $f(x) = \frac{1}{x-3}$  has V.A. at  $x=3$

$$\frac{3x^2 + 3x}{2x^2 + 4x} \approx \frac{3}{2}$$

HORIZONTAL ASYMPTOTES

- ① If  $n < m$  (top degree less than bottom degree), then asymptote is  $y=0$
- ② If  $n = m$  (top degree = bottom degree), then asymptote is the ratio of leading coefficients.
- ③ If  $n > m$  (top degree > bottom degree), then there is no horizontal asymptote

$$\frac{x^3 + 2}{x + 10x}$$

## Examples

$$\textcircled{1} f(x) = \frac{2x+1}{x+1}$$

$$\text{V.A. } x = -1$$

$$\text{H.A. } y = \frac{2}{1}$$

$$\textcircled{2} f(x) = \frac{2x}{3x^2+1}$$

$$\text{V.A.} \rightarrow 3x^2+1=0 \rightarrow x^2 = -\frac{1}{3}$$

$$\text{H.A.} \rightarrow y=0$$

$$x = \text{Non Real}$$

$$\textcircled{3} f(x) = \frac{2x^2}{x^2-1}$$

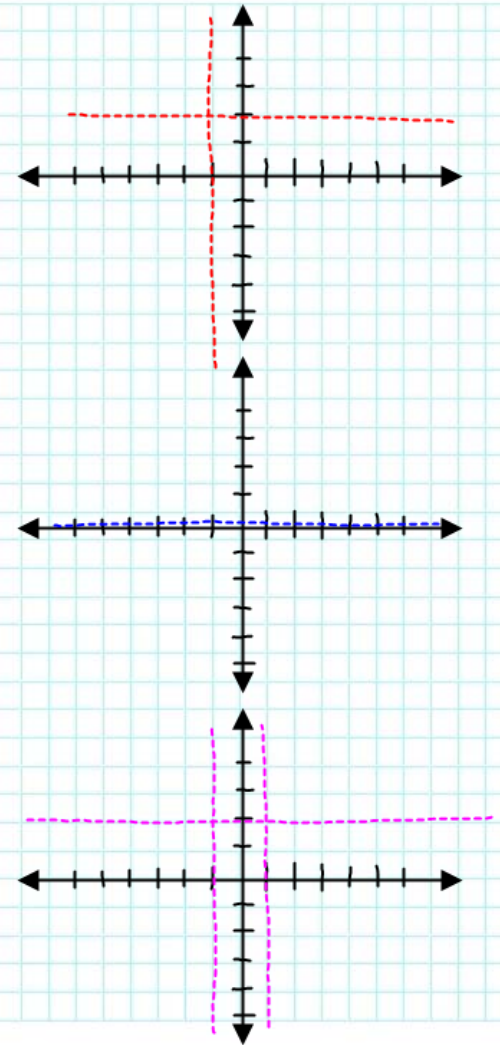
$$x^2-1=0$$

$$x^2=1$$

$$x = \pm 1 \rightarrow \text{V.A.}$$

$$m = 0$$

$$\text{H.A.} = \frac{2}{1}$$

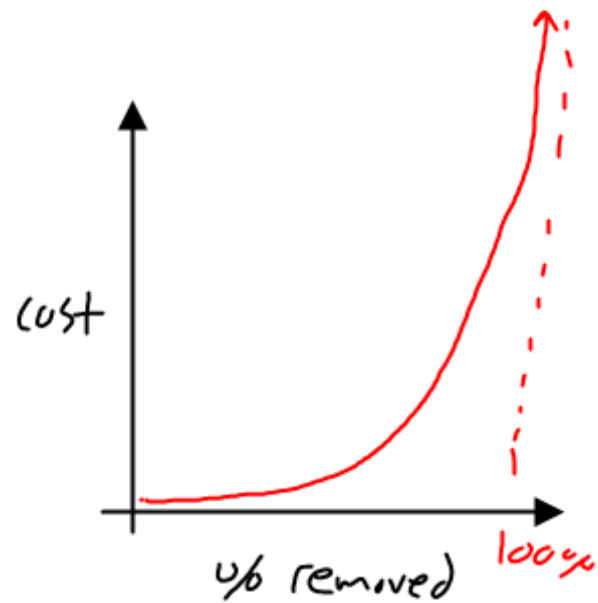


Find the asymptotes  $f(x) = \frac{x^2 + x - 2}{x^2 - x - 6} = \frac{(x-1)(x+2)}{(x-3)(x+2)}$

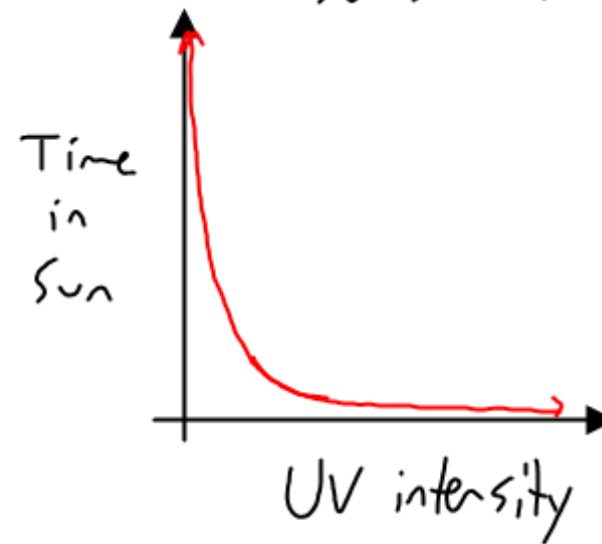
V.A.  $x^2 - x - 6 = 0$   
 $(x-3)(x+2)$   $x = \cancel{-2}, 3$

H.A.  $y = 1$

Cost of cleaning pollutants  
out of a smokestack



The time a person can  
spend in the sun w/o  
burning depends on the  
sun's intensity



## SECTION 2.6

# 1-6 (b+c only), 7-12, 13-21 (odd), 23

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in class